

Physics with Negative Masses

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PACS 03.70.+k - Theory of quantized fields

PACS 04.60.-m - Quantum gravity

Abstract

A correct time translation operator must have an unbounded spectrum. Here we propose that there exists a mass operator with this property that replaces the conventionally used Hamiltonian. Simple consequences are a negative gravitational mass for anti-particles, a zero-energy vacuum and a prospect of benign modifications of quantum field theory, including a quantum theory of gravitation. The advent of negative gravitational mass might help to overcome some of the most fundamental problems in cosmology, but it also questions whether General Relativity is the appropriate theory of gravitation.

Introduction

There is a growing evidence that many of the very basic problems in Cosmology [1] and Physics in general may find a simple explanation by accepting that matter and anti-matter have gravitational mass-values of different sign, i.e. anti-matter has a negative mass [2]. This note is an attempt to dwell on this idea, which has probably been discussed in almost all Theoreticians rooms around the world in the past.

General Remarks

Translational invariance of physical laws in 4-d space-time manifests itself in the conserved energy-momentum 4-vector, $p = (p^0, p^1, p^2, p^3)$. In quantum mechanics p is an operator conjugated to the time-space position operator, x , which is expressed by the commutation relations

$$[x^\mu, p_\nu] = i \delta_\nu^\mu \quad , \quad (1)$$

and the fact that p^μ are the generators of infinitesimal translations of observables, O :

$$\frac{\partial O}{\partial x^\mu} = -i [O, p_\mu] \quad . \quad (2)$$

The Hamiltonian H is conjugated to time and the momentum to position in space. However, as Pauli already pointed out [3], the Hamiltonian is actually an inappropriate choice because its spectrum is bounded from below, i.e. it is essentially a positive operator.

Here we propose to replace H by a mass operator, $p^0 = M$, which can assume unlimited positive and negative values. The states of negative mass values are associated with anti-particle states. For particles, H and M are identical for anti-particles opposite. This may be expressed by

$$H = \text{sign}(M) M \quad , \quad (3)$$

which is to be understood by going to the (common) diagonal representation. Thus, anti-particles are associated with negative-frequency states, an assignment which is invariant under proper orthochronous Lorentz transformations.

Quantum-Field Theoretical Considerations

Standard Quantum Field Theory takes the Hamiltonian as time translation operator, although its bounded spectrum is at variance with the above mentioned fundamental requirement. The boundedness from below is heavily used in the definition of the vacuum state, in the development of scattering theory, in the form of the Feynman propagator etc. Here, we propose a mass operator with unbounded spectrum with the consequence of negative masses for anti-particles. Though we are not in a position to fully develop a quantum field theory with the mass operator as generator of infinitesimal translation in time here, there are a couple of features that can be derived relatively easily and which look very interesting with respect to difficulties with which the standard theory is plagued.

Scalar field

A field has a continuous number of degrees of freedom labeled by the points in 3-d space or equivalently by the points in wave-number (Fourier) k -space. Thus, a free scalar field, ψ , which satisfies the Klein-Gordon equation

$$(\partial^\mu \partial_\mu + m^2) \psi(x) = 0 \quad (4)$$

can be written as a sum over all possible k -vectors. For simplicity we assume a compact x -space and thus an enumerable k -set:

$$\psi(x) = \sum_k^{k^0 = \pm \omega_k} \psi_k e^{-ikx} \quad , \quad (5)$$

$$\text{where } \omega_k = \sqrt{\vec{k}^2 + m^2} \quad . \quad (6)$$

In usual fashion, ψ_k and its adjoint (designated by a hat) are replaced by the annihilation operator

$$a_k = \psi_k + i \hat{\psi}_k \quad (7)$$

and its adjoint creation operator \hat{a}_k . These operators satisfy the commutation relations

$$[a_k, a_l] = 0 \quad , \quad \text{and} \quad [a_k, \hat{a}_l] = \delta_{k,l}^{(4)} \quad . \quad (8)$$

In these variables the mass operator, M , takes the form:

$$M = \sum_k \frac{\omega_k}{2} (a_k \hat{a}_k + \hat{a}_k a_k) \quad . \quad (9)$$

It has exactly the form of the Hamiltonian in the conventional treatment. The difference lies in the definition of the vacuum and correspondingly in the matrix elements of a_k and \hat{a}_k . These annihilation and creation operators, reduce (augment resp.) the number of particles in mode k by one, while the mass operator counts the number of particles of energy ω_k in all modes.

For a single mode one can now start from the vacuum, $|0\rangle$, and build up one-, two- etc. particle states by applying \hat{a} once, twice etc. to the vacuum state ($\hat{a}|0\rangle \rightarrow |1\rangle$, $\hat{a}|1\rangle \rightarrow |2\rangle$, etc). Conventionally, arguing that the energy must be bounded from below, the condition $a|0\rangle = 0$ is requested, i.e. all annihilation operators yield zero when applied to the vacuum.

In contrast to this view, we argue that the mass operator (9) is allowed to assume unlimited negative values. Thus, we consider that a promotes the vacuum $|0\rangle$ to a the state with particle number minus one, i.e. to the state of a single antiparticle, $a|0\rangle \rightarrow |-1\rangle$. In this definition of the vacuum, the operators have the (single mode) matrix elements

$$a_{m,n} = \hat{a}_{n,m} = \sqrt{n + \frac{1}{2}} \delta_{n,m+1} \quad . \quad (10)$$

The number operator $(a\hat{a} + \hat{a}a)/2$ is diagonal with integer eigenvalues, n . The vacuum state is the eigenstate with zero eigenvalue, the state with n particles has eigenvalue n , the one with n antiparticles has eigenvalue $-n$.

It is clear that a_k and \hat{a}_k must be adjoint to each other rather than hermitian conjugate. The latter relationship is connected to a probabilistic (non-negative) interpretation of wave-functions. However, here adjointness allows for either sign, which is interpreted as a mass density which is positive for particles and negative for antiparticles.

The Hamiltonian is simultaneously diagonal with M but, in accordance with (3), the negative eigenvalues of M are to be multiplied by -1 to yield corresponding energy eigenvalues. The vacuum has exactly zero energy and is not plagued with the embarrassingly large zero-point energy that follows from the conventional choice of the vacuum and corresponding matrix elements for the operators a (and \hat{a}) [5].

For a self-adjoint (real) field we have to identify a_k and \hat{a}_{-k} .

General Bosons

The treatment for scalar fields, transforming according to the identity representation of the Lorentz group, carries in principle over to general fields obeying the commutation relations (8), though there are tedious extras owing to the additional degrees of freedom associated with the higher dimensionality of their representations. We will not elaborate on these complications, but for a single remark on photons.

Photons are zero-mass particles that transform under the little group of inhomogeneous Lorentz transformations according to one-dimensional representations characterized by helicity which can take values of ± 1 . The $+1$ and -1 helicity representations are conjugate, which means that we have to identify creation- and annihilation operators according to $\hat{a}^+_{k} = a^-_{-k}$, and correspondingly $\hat{a}_{k} = a^+_{-k}$. Thus, the electromagnetic field is not self-adjoint and there must exist anti-photons [9]. Why do we not see those? The explanation follows from considering the conserved quantities, mass and energy. In a matter island of the universe, two states of a certain energy difference always have a mass difference of the same sign. However, for an anti-photon these signs are opposite. Thus, matter cannot absorb an anti-photon because in such a process mass- and energy-conservation cannot be fulfilled simultaneously.

Fermions

Fermions fields can also be represented in a form analogous to (7), However, the creation and annihilation operators now obey anti-commutation rules:

$$\{a_k, a_l\} = 0 \quad , \quad \text{and} \quad \{a_k, \hat{a}_l\} = \delta_{k,l}^{(4)} \quad . \quad (11)$$

The mass operator for this free field takes the form

$$M = \sum_k \frac{\omega_k}{2} (a_k \hat{a}_k - \hat{a}_k a_k) \quad (12)$$

for a single mode the matrix elements between the states are

$$a_{m,n} = \hat{a}_{n,m} = \sqrt{\frac{1}{2}} (-1)^n \delta_{n,m+1} \quad (13)$$

Now, the number operator $(a\hat{a} - \hat{a}a)/2$ is diagonal with eigenvalues 1, 0, or -1. All states with even n , have eigenvalue zero and are identified with the vacuum. The states with ' n modulo 4' = 1 are identified with the one-particle state, while the ones with ' n modulo 4' = -1 are identified with the one-anti-particle state. Satisfyingly, the vacuum has again no energy.

As in the case of non-scalar Bosons, one has to take care of possible identifications of representations which, however, is rather more delicate here due to the two-valuedness of Fermion representations.

Remark on the Dirac Equation

In the treatment of the Dirac equation

$$-i \partial_t \psi = H \psi = (-i \alpha^k \partial_k + \beta m) \psi \quad (14)$$

the occurrence of negative-energy states was a puzzle for a long time, which was solved in the field-theoretical approach semi-satisfactorily by the introduction of separate sets of creation and annihilation operators for particles and antiparticles. In our interpretation, the generator of time translations in (14) is not to be viewed as the Hamiltonian, H , but as the mass operator, M , that is correctly allowed to assume unbounded negative values. Consequently a single set of “creation” and “annihilation” operators is able to generate particle and anti-particle states as shown in (13).

Path Integrals

In the Dirac-Feynman derivation of the path integral [7] the mass operator, M , now replaces the Hamiltonian as generator of time translations. Correspondingly, the action functional takes the form

$$S = \int dx (\dot{\psi} \hat{\psi} - M(\psi, \hat{\psi})) \quad (15)$$

with the time derivative of the field given by the commutator

$$i \dot{\psi} = [\psi, M] \quad (16)$$

In standard theory where M is replaced by the Hamiltonian with its positive spectrum, convergence of the path integral is considered a limiting process whereby a infinitesimally small imaginary “mass”-term is added to the Hamiltonian to obtain convergent integrals by closing contours in the negative or positive imaginary plane in frequency space, depending on whether a positive or negative value of time is considered. This leads to the use of the Feynman propagator in the evaluation of Feynman diagrams of generic form [7]:

$$D_F(x) = -i \int \frac{d^3 k}{(2\pi)^3 2\omega_k} e^{-i\vec{k}\vec{x}} [\Theta(-x^0) e^{i\omega_k x^0} + \Theta(x^0) e^{-i\omega_k x^0}] \quad (17)$$

where $\Theta(x)$ is the conventional (0,1) step function, and where we have switched to the conventional continuous representation in k -space.

However, when the mass operator M , with its unbounded spectrum, governs time propagation, no closure of any integration path is possible and one has to rely on the assumption that ever faster

oscillations of bounded amplitude lead to convergence of the path integral.

In this situation the following propagator may be more appropriate

$$D_M(x) = i \operatorname{sign}(x^0) \int \frac{d^3 k}{(2\pi)^3 4\omega_k} e^{-i\vec{k}\vec{x}} [e^{i\omega_k x^0} - e^{-i\omega_k x^0}] , \quad (18)$$

which is obtained from taking the principal-value of particle and anti-particle poles on the frequency axis instead of the residue of the positive-frequency pole alone.

Obviously, $D_M(x) = 0$ for $x^0 = 0$. From the Lorentz-invariance of the integral it follows, that $D_M(x) = 0$ for all space-like vectors x , as appropriate for a speed-limited world.

Gravitation Field

We tentatively propose, that the gravitational field is a real mass-less scalar, ϕ . Interaction with other fields is through current coupling with the action integral of the generic form

$$S_{\text{int}} = G \int d^4 x \partial^\mu \phi J_\mu = G \int d^4 x \phi \partial^\mu J_\mu , \quad (19)$$

where J_μ is the current of any field in question and G the gravitational constant. The current of the scalar field is suggested by Lorentz invariance to be of the generic form $\partial_\mu \phi$. Obviously, coupling occurs to any field through the “mass” operator ∂^μ , as we would expect. In addition, the scalar field leads to attraction of like 'charges' [7], which here means matter attracts matter, and anti-matter attracts anti-matter, while matter (positive frequency states) and anti-matter (negative frequency states) repel each other.

In this view, gravitational waves consist of an oscillating scalar, with the gravitational force field pointing along its gradient and, thus, along the wave vector, which corresponds to longitudinal polarization.

Conclusion

We have presented, in a rather sketchy fashion, first steps towards a field theory that leads to a negative gravitational mass of anti-matter. Ideas to attribute a negative mass to anti-matter have been around since its discovery, but have been rejected on the grounds of General Relativity Theory [8], which does not allow for positive and negative curvature in a symmetric fashion. However, there are cosmological riddles of very fundamental nature which would find very simple explanations with the concept of matter-anti-matter repulsion, MAR [2], namely:

- Matter-anti-matter asymmetry (Baryogenesis)
- Horizon problem
- Flatness problem
- Accelerated Expansion (Dark Energy problem)
- Dark Matter problem
- Anti-matter islands must be expected in cosmos, but cannot be seen optically. It is a quite nice speculation that they are located in the huge void regions which make up just about half of all space [9].

Here we suggest in addition that problems in General Physics are made to also disappear with the

concept of a negative mass.

- An unbounded mass operator M provides a proper conjugate to the time operator.
- The Vacuum Energy becomes zero, instead of the large (divergent) values in conventional theory, that cannot be properly accounted for in cosmological models.
- Propagators of the form (18) are exactly zero for space-like distances (compare Ref. 7, p24). Furthermore, they may provide conditionally convergent integrals (of principal-value type) for loop diagrams instead of divergent ones which have to be kept finite by small-scale cutoffs and renormalization.

It is clear, that our proposal poses more questions than it answers in this paper. There is a host of situations which will have to be reconsidered in this light. However, the prospect of getting answers to the above list of fundamental problems certainly makes it worthwhile to follow up on this approach.

References

Since this work is mostly a reinterpretation of textbook knowledge, citation is restricted to the few standard texts which have been chiefly consulted [4-7].

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